

correctly that the negative experimental results could have been anticipated, had their stability analysis been performed in advance. The analysis showed that the effect of localized heating concentrated at the nose is different from the more common effect of heat transfer that is distributed over a large area, and hence different transition behavior can be expected.

The negative experimental results also could have been anticipated from the results of previous investigations. In 1942, Frick and McCullough² measured the variation in transition Reynolds number due to localized heating of various portions of a low-drag airfoil. They found that heating the upper surface of the airfoil caused an inflection point in the velocity profile, as expected, and a subsequent reduction in transition Reynolds number. However, when only the nose was heated, corresponding to Cebeci and Smith's case, there was no inflection point in the boundary layer far downstream and no change in transition Reynolds number.

More recently, McCroskey and Lam³ found that heat applied at the leading edge of a flat plate produces no inflection point in the theoretical velocity profiles downstream, if the wall is adiabatic downstream. This result contrasts importantly with the well-known result that more uniform heating along a surface produces an inflection point and destabilizes the laminar boundary layer. In the experimental part of their investigation, McCroskey and Lam found that the isolated heat source at the origin actually delayed transition rather than promoting the onset of turbulence.

On the basis of the aforementioned results, it is, therefore, not surprising that Cebeci and Smith did not obtain earlier transition by heating the leading edge.

References

- ¹ Cebeci, T. and Smith, A. M. O., "Investigation of Heat Transfer and of Suction for Tripping Boundary Layers," *Journal of Aircraft*, Vol. 5, No. 5, Sept.-Oct. 1968, pp. 450-454.
- ² Frick, C. W., Jr. and McCullough, G. B., "Tests of a Heated Low-Drag Airfoil," Wartime Report A-40, 1942, NACA.
- ³ McCroskey, W. J. and Lam, S. H., "The Temperature-Vorticity Analogy in Boundary Layers," *International Journal of Heat Mass Transfer*, Vol. 9, No. 11, Nov. 1966, pp. 1205-1217.

Reply by Authors to W. J. McCroskey

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THE authors read Mr. McCroskey's comments with a mixture of pleasure and chagrin. The pleasure comes from the general agreement between their tests and the NACA Ames tests¹ cited by McCroskey. The chagrin comes from not finding Ref. 1. In researching the literature prior to the said experiment, the authors encountered Ref. 1; upon requesting this reference from their library, Ref. 2 was promptly obtained, a reference which has exactly the same report number, the same issue date, and the same authors as Ref. 1! Two thorough searches, by their library, through the entire Los Angeles area for Ref. 1 proved futile. The NACA Ames library verified that indeed the two reports were issued with the same number (Ref. 2 has subsequently been reissued as NACA TR 830). Thus, after a

lengthy period of unsuccessful attempts to obtain Ref. 1, the authors gave up.

A few comments are in order about the tests of Ref. 1. These tests appear to be good and contain data that should be valuable to fundamental studies of the transition process. It does not deserve to be buried as an NACA Wartime Report. In this report, temperature differences of only 100°F were used. The authors tried much higher temperature differences, over 400°F, but the results were still negative. However, they did succeed in relating the experimental results to theory much more completely than in Ref. 1.

References

- ¹ Frick, C. W., Jr. and McCullough, G. B., "Tests of a Heated Low-Drag Airfoil," Wartime Report A-40, 1942, NACA.
- ² Frick, C. W., Jr. and McCullough, G. B., "A Method for Determining the Rate of Heat Transfer from a Wing or Streamline Body," Wartime Report A-40, 1942, NACA.

Comment on "Strength Margins for Combined Random Stresses"

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Introduction

IN Ref. 1, the problem of finding the average frequency of crossing of a curve in the (x, y) plane by a Gaussian stationary random vector $\mathbf{z}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$ is considered. Two of the correlation coefficients defining the process were assumed to be zero. We will include all correlation coefficients and examine the problem where the curve can be conveniently approximated by straight line segments.

Average Crossing Frequency of a Straight Line Segment

Consider a straight line segment in the (x, y) plane from (x_1, y_1) to (x_2, y_2) . Let (s, n) be orthogonal coordinates parallel to and normal to the segment respectively, with the same origin 0 as (x, y) . In the (s, n) coordinates the segment lies between (s_1, n_1) and (s_2, n_2) . Using the same procedure as in Ref. 1, we find the average crossing frequency per unit length of segment in the positive n direction ($\dot{n} > 0$),

$$\bar{N}_{cp}(s, n) = \int_0^\infty \int_{-\infty}^\infty p(s, n, \dot{s}, \dot{n}) d\dot{s} d\dot{n} \quad (1)$$

where $p(s, n, \dot{s}, \dot{n})$ is the probability density function of position and velocity of the vector $\mathbf{z}(t)$ in the (s, n) coordinates and $(\dot{\cdot})$ represents differentiation with respect to time. The corresponding average for crossings in the negative n direction ($\dot{n} < 0$) is

$$\bar{N}_{cn}(s, n) = \int_{-\infty}^0 \int_{-\infty}^\infty p(s, n, \dot{s}, \dot{n}) d\dot{s} (-\dot{n}) d\dot{n} \quad (2)$$

which, in general, is not the same as $\bar{N}_{cp}(s, n)$.

We now integrate to obtain the average frequency of crossing of the line segment in the positive n direction, say

$$N_{cp}(s_1, s_2, n_1) = \int_{s_1}^{s_2} \bar{N}_{cp}(s, n_1) ds \quad (3)$$

This compares with Eq. (11) of Ref. 1, where crossings in both directions are considered together, and a general curve C is taken instead of a straight line segment. In Ref. 1,

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